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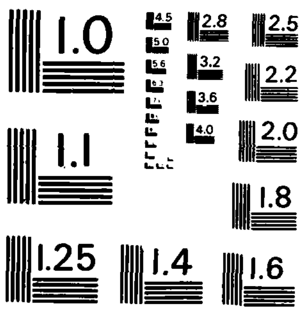

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Rough Surface Scattering via the Smoothing Method*

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Abstract. The smoothing method is used to find the first two moments, *i.e.*, the mean and the two-point two-time correlation function, of the field scattered by a rough surface. The results are expressed in terms of a reflection coefficient and a differential scattering coefficient. They are compared with those found by several other methods.

1. Introduction

An acoustic wave which hits a rough surface produces a mean or coherent reflected field and a fluctuating or partially coherent scattered field. These fields can be calculated by the regular perturbation or Born expansion method, see *e.g.*, [1], but the results diverge at grazing incidence. However they can be modified or renormalized to remain finite [1] by writing them in forms suggested by Twersky's self-consistent field method [2, 3]. We shall show that these same non-divergent results can be obtained directly by a different perturbation procedure, known as the smoothing method.

The smoothing method has been used to find the coherent field for certain kinds of rough surfaces by Wenzel [4], DeSanto, and others. (See DeSanto's review [5].) Here we apply it to more kinds of rough surfaces, which can be either moving or at rest, and we calculate both the coherent field and the two-point two-time correlation function of the field. Then we show that the results are the same as the renormalized ones in [1], which were compared there with Twersky's [2, 3] results for embossed surfaces. Thus the present calculation elucidates the relationship among the results of the regular perturbation or Born expansion method, the renormalized regular perturbation method, Twersky's self-consistent field method and the smoothing method.

2. The Integral Equation for the Scattered Field

Suppose that a wave with the acoustic velocity potential $\psi(x,y,z,t)$ is incident upon a rough surface S, and that it produces a scattered wave with potential $\varphi(x,y,z,t)$. Both potentials must satisfy the wave equation with constant sound speed c above the surface S, and φ must be outgoing. In addition $\psi+\varphi$ must satisfy on S a boundary condition which depends upon the nature of the surface. We shall consider the following four boundary conditions:

$$\partial_t(\psi+\varphi)=0 \quad \text{on } z=\varepsilon h \tag{1a'}$$

$$(\partial_z-\varepsilon h_x\partial_x-\varepsilon h_y\partial_y)(\psi+\varphi)=\varepsilon h_t \quad \text{on } z=\varepsilon h \tag{1b'}$$

$$(\varepsilon h\partial_t+c\partial_z)(\psi+\varphi)=0 \quad \text{on } z=0 \tag{1c}$$

$$(\partial_t+\varepsilon ch\partial_z)(\psi+\varphi)=0 \quad \text{on } z=0 \tag{1d}$$

In all cases ε is a small parameter and $h(x,y,t)$ is a given function. Case a represents a soft surface, $z=\varepsilon h$, on which the pressure vanishes; case b represents a hard surface $z=\varepsilon h$ on which the acoustic normal velocity

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The smoothing method is used to find the first two moments, i.e:, the mean and the two-point two-time correlation function, of the field scattered by a rough surface. The results are expressed in terms of a reflection coefficient and a differential scattering coefficient. They are compared with those found by several other methods.

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equals the surface normal velocity, case c represents a flat surface $z = 0$ with the admittance ϵh, and case d represents a flat surface $z = 0$ with the impedance ϵh. In case b we shall not consider the acoustic radiation produced by the motion of the surface, so we shall omit the term ϵh_t on the right side of (1b').

Before proceeding we shall simplify (1a') and the homogeneous form of (1b') by expanding $\psi + \varphi$ in a Taylor series about $z = 0$ to obtain

$$[1 + \epsilon h\partial_z + \frac{\epsilon^2 h^2}{2}\partial_z^2 + O(\epsilon^3)]\partial_t(\psi + \varphi) = 0 \quad \text{on } z = 0 \tag{1a}$$

$$[\partial_z + \epsilon(h\partial_z^2 - h_x\partial_x - h_y\partial_y) + \epsilon^2(\frac{h^2}{2}\partial_z^3 - hh_x\partial_x\partial_z - hh_y\partial_y\partial_z) + O(\epsilon^3)](\varphi + \psi) = 0 \quad \text{on } z = 0 \tag{1b}$$

Now all four boundary conditions (1a)–(1d) apply on the plane $z = 0$. This makes it convenient to introduce the Fourier transform pairs:

$$F(s) = (2\pi)^{-3}\int f(p)e^{-ip\cdot s}dp \tag{2}$$

$$f(p) = \int F(s)e^{ip\cdot s}ds \tag{3}$$

Here $p = (x, y, t)$ and $s = (\alpha, \beta, \omega)$ are three component vectors in physical space-time and in wavenumber-frequency space, respectively. We shall always use the corresponding capital letter to denote the transform of a function denoted by a small letter.

We now apply the transform (2) to the wave equation for φ and find that $\Phi(s, z)$ satisfies the ordinary differential equation $\Phi_{zz} + k^2(s)\Phi = 0$ where $k^2(s) = \omega^2/c^2 - \alpha^2 - \beta^2$. Since Φ must be outgoing at $z = +\infty$ when k is real, and must vanish there when k is imaginary, Φ must be of the form

$$\Phi(s, z) = A_s(s)e^{-ik(s)z} \tag{4}$$

Here $k(s)$ is defined by

$$k(s) = (\omega/c)[1 - (\alpha^2 + \beta^2)(\omega/c)^{-2}]^{\frac{1}{2}} \quad , \quad (\omega/c)^2 \geq \alpha^2 + \beta^2 \tag{5}$$

$$k(s) = -i[\alpha^2 + \beta^2 - (\omega/c)^2]^{\frac{1}{2}} \quad , \quad (\omega/c)^2 \leq \alpha^2 + \beta^2$$

The scattered amplitude $A_s(s)$ is unknown, and is to be found. To find it we first write $\Psi(s, z)$, the transform of the incident field, as

$$\Psi(s, z) = A_i(s)e^{ik(s)z} \tag{6}$$

The incident amplitude $A_i(s)$ is assumed to be known. We now apply the transform (2) to each of the boundary conditions (1a) - (1d) and use (4) and (6) in the resulting equations. In each case we obtain an integral equation for $A_s(s)$ of the form

$$K(-\epsilon)A_s = \pm K(\epsilon)A_i \tag{7}$$

The linear operator $K(\epsilon)$ in (7), which depends upon ϵ, can be written as

$$K(\epsilon) = I + \epsilon K_1 + \epsilon^2 K_2 + O(\epsilon^3) \tag{8}$$

Here I is the identity while K_1 and K_2 are integral operators of the forms

$$K_1 A = \int H(s - s')m(s, s')A(s')ds' \tag{9}$$

$$K_2 A = -\frac{1}{2}\iint H(s - s')H(s' - s'')\left[\frac{\omega''}{\omega}\right]k^2(s'')A(s'')ds''ds' \tag{10a}$$

$$K_2 A = \frac{1}{2} \iint H(s-s')H(s'-s'')\frac{k(s'')}{k(s)}\left\{k^2(s'') - 2[(\omega''/c)^2 - \alpha'\alpha'' - \beta'\beta'']\right\}A(s'')ds''ds' \quad (10b)$$

We recall that H is the transform of h, while m and the choice of sign in (7) are given in Table I. For the admittance and impedance boundary conditions, cases c and d, the operator K_2 and the $O(\varepsilon^2)$ term in (8) are both zero.

Parameter	(a) Soft	(b) Hard	(c) Admittance	(d) Impedance
$\pm$	$-$	$+$	$+$	$-$
$m(s,s')$	$i\omega' k(s')/\omega$	$ik^{-1}(s)\left[\left(\frac{\omega'}{c}\right)^2 - \alpha\alpha' - \beta\beta'\right]$	$\omega'/ck(s)$	$\omega/ck(s')$

Table I. The sign and the function $m(s,s')$.

3. Application of the Smoothing Method

Let h and A_i be random functions statistically independent of one another. Then the solution A_s of the integral equation (7) is random also. To calculate its first two moments we shall use the smoothing method, first used to treat waves in random media and similar problems by Bourret [6] and Keller [7]. It is convenient to write $\langle f \rangle$ to denote the mean value with respect to h of any random function f that depends upon h, and $f' = f - \langle f \rangle$ to denote the corresponding fluctuating part of f. Then $\langle f' \rangle = 0$. If f depends upon both h and A_i, then $\langle f \rangle$ still depends upon A_i, so it is still random. We denote the average of f with respect to both h and A_i by $\langle\langle f \rangle\rangle$.

We begin by using (8) in (7) to get

$$(I - \varepsilon K_1 + \varepsilon^2 K_2)A_s = \pm(I + \varepsilon K_1 + \varepsilon^2 K_2)A_i + O(\varepsilon^3) \quad (11)$$

Now we take the mean value of (11) with respect to h to obtain

$$(I - \varepsilon\langle K_1\rangle + \varepsilon^2\langle K_2\rangle)\langle A_s\rangle - \varepsilon\langle K_1' A_s'\rangle + \varepsilon^2\langle K_2' A_s'\rangle = \pm(I + \varepsilon\langle K_1\rangle + \varepsilon^2\langle K_2\rangle)A_i + O(\varepsilon^3) \quad (12)$$

Next we subtract (12) from (11), keeping terms through $O(\varepsilon)$, to get

$$A_s' - \varepsilon(K_1'\langle A_s\rangle + \langle K_1\rangle A_s' - K_1' A_s' + \langle K_1' A_s'\rangle) = \pm\varepsilon K_1' A_i + O(\varepsilon^2) \quad (13)$$

Upon solving (13) for A_s' up to $O(\varepsilon)$, we find

$$A_s' = \varepsilon K_1'\langle A_s\rangle \pm \varepsilon K_1' A_i + O(\varepsilon^2) \quad (14)$$

Finally we use (14) for A_s' in (12) and obtain

$$[I - \varepsilon\langle K_1\rangle - \varepsilon^2\langle K_1' K_1'\rangle + \varepsilon^2\langle K_2\rangle]\langle A_s\rangle = \pm[I + \varepsilon\langle K_1\rangle + \varepsilon^2\langle K_1' K_1'\rangle + \varepsilon^2\langle K_2\rangle]A_i + O(\varepsilon^3) \quad (15)$$

This is the desired smoothed or non-random equation for $\langle A_s\rangle$, and (14) is an expression for A_s' in terms of $\langle A_s\rangle$. The operators $\langle K_1\rangle$ and $\langle K_2\rangle$ in (15) are given respectively by (9) with H replaced by $\langle H\rangle$ and by (10) with HH replaced by $\langle HH\rangle$. The operator $\langle K_1' K_1'\rangle$ is given by

$$\langle K_1' K_1'\rangle A = \int \langle H'(s-s')H'(s-s'')\rangle m(s,s')m(s',s'')A(s'')ds'ds'' \quad (16)$$

The integral equation (15) simplifies considerably when the function h is statistically second order stationary. Then the mean of h is a constant h_o, i.e., $\langle h \rangle = h_o$, and the two-point two-time correlation function of h depends only upon the difference between its arguments, i.e. $\langle h(p+p')h(p')\rangle = r(p) + h_o^2$. Consequently the first two moments of the transform H are given by $\langle H(s)\rangle = h_o\delta(s)$ and $\langle H(s)H(s')\rangle = R(s)\delta(s+s') + h_o^2\delta(s)\delta(s')$. Here $R(s)$, the spectral power density of $h(p)$, is the Fourier transform of the auto-correlation function $r(p)$, so it is a positive, even, real-valued function of s. When the expressions for $\langle H\rangle$ and $\langle H(s)H(s')\rangle$ are used in the average operators, they simplify to,

$$\langle K_1\rangle A = h_o m(s,s)A(s) \tag{17}$$

$$\langle K_1'K_1'\rangle A = \left[\int R(s-s')m(s,s')m(s',s)ds'\right]A(s) \tag{18}$$

$$\langle K_2\rangle A = -\tfrac{1}{2}k^2(s)[V + h_o^2]A(s) \tag{19a,b}$$

Here $V = \int R(s)ds$ is the variance of h. Thus for stationary processes, these three operators are multiplicative, and the smoothed equation (15) is just a linear algebraic equation for $\langle A_s\rangle$. In cases c and d, $\langle K_2\rangle = 0$

4. Solution for the Mean Amplitude

In general the solution of (15) for $\langle A_s\rangle$ can be written

$$\langle A_s\rangle = MA_i + O(\epsilon^3) \tag{20}$$

The linear operator M, which occurs in (20), is not random, and must be found by solving the integral equation (15). However, when $h(p)$ is second order stationary, the solution of (15) is

$$\langle A_s(s)\rangle = C(s)A_i(s) + O(\epsilon^3) \tag{21}$$

Here $C(s)$ is just a scalar function which we call the reflection coefficient. It is given by

$$C(s) = \pm[1 + \epsilon\langle K_1\rangle + \epsilon^2\langle K_1'K_1'\rangle + \epsilon^2\langle K_2\rangle]/[1 - \epsilon\langle K_1\rangle - \epsilon^2\langle K_1'K_1'\rangle + \epsilon^2\langle K_2\rangle] + O(\epsilon^3) \tag{22}$$

$$= \pm[1 + Q(s)]/[1 - Q(s)] + O(\epsilon^3)$$

The sign is shown in Table I and $Q(s)$ is defined by

$$Q(s) = \left[\epsilon h_o m(s,s) + \epsilon^2\int R(s-s')m(s,s')m(s,s')ds'\right]\left[1 - \frac{\epsilon^2}{2}k^2(s)\left(V + h_o^2\right)\right]^{-1} \tag{23a,b}$$

$$Q(s) = \epsilon h_o m(s,s) + \epsilon^2\int R(s-s')m(s,s')m(s,s')ds' \tag{23c,d}$$

When (21) is averaged with respect to A_i, it yields

$$\langle\langle A_s(s)\rangle\rangle = C(s)\langle A_i\rangle + O(\epsilon^3) \tag{24}$$

Here the average of A_s pertains to both h and A_i, while $\langle A_i\rangle$ means the average of A_i, which is independent of h.

The second form of $C(s)$ in (22) is obtained from the first form by dividing numerator and denominator by $1 + \epsilon^2\langle K_2\rangle$, following Twersky [8]. If the denominator in (23a,b) is replaced by unity, which corresponds to an $O(\epsilon^3)$ change in $C(s)$, the resulting $C(s)$ is identical with equation (42) of [1], which is the renormalised Born reflection coefficient.

5. The Second Moment of the Field

Let us consider the total acoustic potential $\psi + \varphi$ at two points with separation $x = (x, y, z, t)$ and midpoint $X = (X, Y, Z, T)$. The second moment or two-point two-time correlation function of the acoustic potential at these points is $\gamma(x, X) = \langle\langle[\psi + \varphi](X + \frac{x}{2})[\psi + \varphi](X - \frac{x}{2})\rangle\rangle$. We denote its Fourier transform $\Gamma(s, q, z, Z)$ where $s = (\alpha, \beta, \omega)$ corresponds to (x, y, t) and $q = (a, b, w)$ corresponds to (X, Y, T), and we write it as

$$\Gamma(s, q, z, Z) = \langle\langle[\Psi + \Phi](\frac{q}{2} + s, Z + \frac{z}{2})[\Psi + \Phi](\frac{q}{2} - s, Z - \frac{z}{2})\rangle\rangle \tag{25}$$

By using (4) and (6) for Φ and Ψ we can express Γ in terms of the four correlations $B_{ii}(s, q) = \langle\langle A_i(\frac{q}{2} + s)A_i(\frac{q}{2} - s)\rangle\rangle$, $B_{is} = \langle\langle A_i A_s\rangle\rangle$, $B_{si} = \langle\langle A_s A_i\rangle\rangle$ and $B_{ss}(s, q) = \langle\langle A_s(\frac{q}{2} + s)A_s(\frac{q}{2} - s)\rangle\rangle$. See [1].

Each of the four correlations above can be evaluated by writing $A_s = \langle A_s\rangle + A'_s$ with $\langle A_s\rangle$ given by (20) and A'_s given by (14). In particular the auto-correlation of the scattered amplitude A_s becomes

$$B_{ss}(s, q) = \langle\langle A_s(\frac{q}{2} + s)\rangle\langle A_s(\frac{q}{2} - s)\rangle\rangle + \langle\langle A'_s(\frac{q}{2} + s)A'_s(\frac{q}{2} - s)\rangle\rangle + O(\varepsilon^3) \tag{26}$$

$$B_{ss}(s, q) = \langle[MA_i](\frac{q}{2} + s)[MA_i](\frac{q}{2} - s)\rangle + \varepsilon^2\langle\langle[K'_1(M \pm I)A_i](\frac{q}{2} + s)[K'_1(M \pm I)A_i](\frac{q}{2} - s)\rangle\rangle + O(\varepsilon^3)$$

When h is second order stationary, M becomes multiplication by $C(s)$, and (26) simplifies to

$$B_{ss}(s, q) = C(\frac{q}{2} + s)C(\frac{q}{2} - s)B_{ii}(s, q) + 4\varepsilon^2 \int \frac{R(s - s')m(\frac{q}{2} + s, \frac{q}{2} + s')m(\frac{q}{2} - s, \frac{q}{2} - s')B_{ii}(s', q)}{[1 - Q(\frac{q}{2} + s')][1 - Q(\frac{q}{2} - s')]} ds' + O(\varepsilon^3) \tag{27}$$

The auto-correlation B_{ss} simplifies even more when the incident field $\psi(x, y, z, t)$ is statistically second order stationary in x, y and t. Then B_{ii} is just

$$B_{ii}(s, q) = I_i(s)\delta(q) \tag{28}$$

where $I_i(s)$ is the intensity of the incident field. With (28) used in it, (27) becomes

$$B_{ss}(s, q) = I_s(s)\delta(q) + O(\varepsilon^3) \tag{29}$$

The function $I_s(s)$ in (29), which is the intensity of the outgoing field in the direction s, is defined by

$$I_s(s) = |C(s)|^2 I_i(s) + |k^{-2}(s)| \int \sigma(s, s')I_i(s')ds' + O(\varepsilon^3) \tag{30}$$

The coefficient $\sigma(s, s')$ in (30) can be identified as the differential scattering coefficient of the surface. It is defined by

$$\sigma(s, s') = \frac{4\varepsilon^2|k^2(s)|R(s - s')|m^2(s, s')|}{|1 - Q(s')|^2} \tag{31}$$

With the denominator in (23a,b) replaced by unity, this is exactly the result for σ given in equation (44) of [1], and called there the renormalized Born approximation.

6. Conclusion

By using the smoothing method, we have calculated the first two moments of the field produced when a possibly random incident field hits a random slightly rough surface. The first moment of the field, called the coherent field, consists of an incident and a reflected field. The reflected field is determined by a reflection coefficient $C(s)$, which depends upon the first two moments of the surface roughness h. The second moment of the field involves a differential scattering coefficient $\sigma(s, s')$ which also depends upon the second moment of h.

Our results for $C(s)$ and $\sigma(s, s')$ are the same as those we obtained before [1] by renormalizing the divergent results given by the Born approximation. That renormalization was achieved by writing C and σ in the forms obtained by Twersky [2,3]. We also showed in [1] that for an embossed plane, our results are the same as his except for one difference. We had the second Born approximation to the differential scattering amplitude of a single boss instead of the exact amplitude which occurs in his theory. This same comparison applys to the results of the smoothing method, as used here.

We conclude that three different methods yield the same results for the C and σ of an embossed plane, provided that each boss is small enough, with a small enough slope, so that its differential scattering amplitude is well approximated by its second Born approximation. The three methods are Twersky's method, the renormalized Born approximation and the smoothing method as used here. When the bosses or their slopes are not small enough, Twersky's results are better than the other two. On the other hand, if the surface is not an embossed plane, Twersky's method does not apply but the other two methods do.

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